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Study of the linked dipole chain model in heavy quark production at the Tevatron

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Abstract: We present calculations of charm and beauty production at Tevatron within the framework of k_T-factorization, using the unintegrated gluon distributions as obtained from the Linked Dipole Chain model. The analysis covers transverse momentum and rapidity distributions and the azimuthal correlations between b and $\bar{b}$ quarks (or rather muons from their decay) which are powerful tests for the different unintegrated gluon distributions. We compare the theoretical results with recent experimental data taken by DØ and CDF collaborations at the Tevatron Run I and II.

Keywords: QCD, Jets, Phenomenological Models, Hadronic Colliders.

Contents

1. Introduction

It is known that in the description of a given cross section in lepton-proton or proton-proton interactions at high energies it is not enough to consider only the leading order perturbative terms. Although at large scales, μ, the running coupling constant α_s may be small, each power of α_s is accompanied by large logarithms due to the large phase space available for the additional gluon radiation. The solution of this problem is to resum the leading logarithmic behavior of the cross section to all orders, thus rearranging the perturbative expansion into a more rapidly converging series.

The most familiar resummation strategy is based on Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [1]–[4] evolution equation, where large logarithmic terms proportional $\alpha_s^n \ln^n(\mu^2/\Lambda^2_{\rm QCD})$ are taken into account. The cross sections can be rewritten in terms of process-dependent hard matrix elements convoluted with universal parton density functions which are described by the DGLAP equation. In this way the dominant contributions come from diagrams where parton emissions in initial state are strongly ordered in virtuality. This is called collinear factorization, as the strong ordering means that the virtuality of the parton entering the hard scattering matrix elements can be neglected compared to the large scale μ.

DGLAP evolution describes most experimental results from electron-proton and proton-proton colliders. By using input parton densities which are sufficiently singular when $x \to 0$, this formalism can also account for the strong rise of F_2 at small x, as observed at HERA. However, there are problems with the description of non-inclusive observables such as forward jet production in ep and heavy quark production in ep and $p\bar{p}$ collisions.

At the energies of the HERA, Tevatron and LHC colliders, the hard scale μ of the heavy quark and quarkonium production processes is large compare to the $\Lambda_{\rm QCD}$ parameter but on the other hand μ is much less than the total center-of-mass energy: $\Lambda_{\rm QCD} \ll \mu \ll \sqrt{s}$. Therefore in such case it was expected that the DGLAP evolution should break down. The situation is classified as "semihard".

It is believed that at asymptotically large energies (very small $x \propto \mu^2/s$) the theoretically correct description is given by the Balitsky-Fadin-Kuraev-Lipatov(BFKL) [5, 6, 7] evolution equation. Here large terms proportional to $\alpha_s^n \ln^n(1/x)$ are taken into account. The BFKL evolution equation actually predicts a strong power-like rise of F_2 at small x. Just as for DGLAP, it is possible to factorize an observable into a convolution of process-dependent hard matrix elements with universal parton distributions. But as the virtualities (and transverse momenta) of the propagating gluons are no longer ordered, the matrix elements have to be taken off-shell and the convolution made also over transverse momentum with the unintegrated gluon distribution $\Phi(x, k_T^2)$. The unintegrated gluon distribution determines the probability to find a gluon carrying the longitudinal momentum fraction x and transverse momentum k_T. This generalized factorization is called k_T-factorization [8, 9].

The unintegrated gluon distribution in the proton is a subject of intensive studies [10]. Knowledge of these is in particular necessary for the description of heavy quark production at future colliders, such as the LHC. This quantity depends on more degrees of freedom than the usual collinear parton density, and is therefore less constrained by the experimental data. Various approaches to model the unintegrated gluon distribution have been proposed. One such approach, valid for both small and large x, has been developed by Ciafaloni, Catani, Fiorani and Marchesini, and is known as the CCFM model [11]–[14]. It introduces angular ordering of emissions to correctly treat gluon coherence effects. In the limit of asymptotic energies and small, fixed coupling, it is almost equivalent to BFKL [15, 16, 17], but also similar to the DGLAP evolution for large x and high μ^2. The resulting unintegrated gluon distribution functions depend on two scales, the additional scale $\bar{q}$ being a variable related to the maximum angle allowed in the emission.

The Linked Dipole Chain Model (LDC) [18, 19] is a reformulation and generalization of the CCFM model, and agrees with CCFM to leading double logarithmic accuracy. Also LDC is formulated in terms of k_T-factorization and unintegrated distribution functions, but here these distributions are essential single-scale dependent quantities. In this paper we will apply the LDC formalism for the description of the charm and bottom production processes at Tevatron.

The application of k_T-factorization supplemented with the BFKL and CCFM evolution equations to heavy quark hadroproduction is discussed in [20]–[27]. It was shown that $b\bar{b}$ production cross section at Tevatron can be consistently described using the k_T-factorization approach together with different BFKL or CCFM-like unintegrated gluon distributions. The NLO pQCD calculations based on the DGLAP evolution scheme underestimate data by a factor about 2–5 [28]–[32]. In a previous paper [27] the dependences of the b quark, B meson and their decay muon cross sections on different forms of the unintegrated gluon distribution was investigated. It was found that the properties of different unintegrated gluon distributions strongly influences the $b\bar{b}$ or muon-muon azimuthal correlations since these quantities are sensitive to the relative contributions of different production mechanisms to the total cross section [20]–[25].

Based on the above mentioned results here we will use the k_T-factorization approach together with LDC unintegrated gluon distribution functions [33] for the analysis of the experimental data [28]–[32]. To illuminate the effect of the different contributions we will study three different versions of LDC unintegrated gluon distributions presented in [33]. It is interesting to note that the LDC unintegrated gluon distributions has been fitted to the inclusive F_2 data at HERA, and for heavy quark hadroproduction at Tevatron we will obtain essentially parameter-free theoretical results. We also present our predictions for the differential cross section of D^*, D^+ and D^0 meson production in $p\bar{p}$ collisions at Tevatron Run II [34].

This article is organized as follows. In section 2 we give a short review of the CCFM and LDC formalism. In section 3 we present the numerical results of our calculations and compare them with the DØ [31, 32] and CDF [28, 29, 30, 34] data. Finally, in section 4, we give some conclusions.

2. The CCFM evolution and LDC model

In figure 1 we show a typical fan diagram for the initial-state radiation in a DIS lepton-proton event. According to the CCFM formalism the emission of gluons during the initial state cascade is only allowed in an angular-ordered region of phase space. In addition the initial state emissions are ordered in the positive (along incoming proton) light-cone momentum k_+. All other kinematically allowed emissions (symbolized by q_1' emission in figure 1) are defined as final-state emissions.

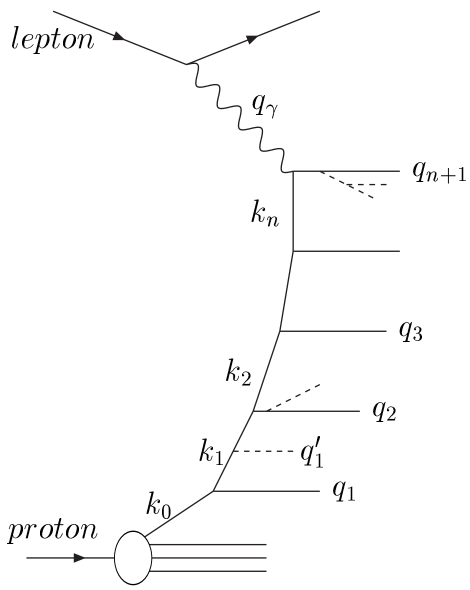

Figure 1: A fan diagram for a DIS event. The quasi-real partons from the initial-state radiation are denoted q_i, and the virtual propagators k_i. The dashed lines denote final-state radiation.

The CCFM evolution equation can be written in a differential form [11]–[14]:

$$\bar{q}^2 \frac{d}{d\bar{q}^2} \frac{\Phi(x, k_T^2, \bar{q}^2)}{\Delta_s(\bar{q}^2, Q_0^2)} = \tag{2.1}$$
$$= \int dz \frac{d\phi}{2\pi} \frac{\tilde{P}_g(z, (\bar{q}/z)^2, k_T^2)}{\Delta_s(\bar{q}^2, Q_0^2)} \, \Phi\left(x', k_T^{2\,\prime}, \left(\frac{\bar{q}}{z}\right)^2\right)$$

where $\Phi(x, k_T^2, \bar{q}^2)$ is the unintegrated gluon distribution, $\vec{k}_T = \vec{k}_T' + \vec{q}_T$, and $\bar{q} = q_T/(1-z)$ is the rescaled transverse momentum of an emitted gluon (with azimuthal angle ϕ). The Sudakov form factor Δ_S is given by:

$$\ln \Delta_s(\bar{q}^2, Q_0^2) = -\int\limits_{Q_0^2}^{\bar{q}^2} \frac{dq^2}{q^2} \int\limits_0^{1-Q_0/q} dz \frac{\bar{\alpha}(q^2(1-z)^2)}{1-z} \tag{2.2}$$

with $\bar{\alpha} = 3\alpha_S/\pi$. For inclusive quantities at leading logarithmic order, the Sudakov form factor cancels the $1/(1-z)$ collinear singularity of the splitting function. The splitting function $\tilde{P}_g$ for branching i is given by:

$$\tilde{P}_g(z_i, q_i^2, k_{Ti}^2) = \frac{\bar{\alpha}(q_i^2(1-z_i)^2)}{1-z_i} + \frac{\bar{\alpha}(k_{Ti}^2)}{z_i} \Delta_{\rm ns}(z_i, q_i^2, k_{Ti}^2) \tag{2.3}$$

where the non-Sudakov (or non-eikonal) form factor $\Delta_{\rm ns}$ is defined as:

$$\ln \Delta_{\rm ns}(z_i, q_i^2, k_{Ti}^2) = -\int\limits_{z_i}^{1} \frac{dz'}{z'} \int \frac{dq^2}{q^2} \bar{\alpha} \theta(k_{Ti} - q) \theta(q - z' q_i) \,. \tag{2.4}$$

The CCFM equation incorporates both BFKL and DGLAP evolution. In the small x limit the unintegrated gluon distribution obeying CCFM evolution equation (2.1) can be written as:

$$\Phi(x, k_T^2, \bar{q}^2) \sim \sum_n \int \prod^n \bar{\alpha} \frac{dz_i}{z_i} \frac{d^2 q_{Ti}}{\pi q_{Ti}^2} \Delta_{\rm ne}(z_i, k_{Ti}^2, \bar{q}_i^2) \times$$
$$\times \delta\left(x - \prod z_i\right) \theta(\bar{q}_i - \bar{q}_{i-1} z_{i-1}) \delta(k_T^2 - k_{Tn}^2) \theta(\bar{q} - \bar{q}_n z_n) \tag{2.5}$$

where the splitting parameter z is defined as $z_i = k_{+,i}/k_{+,i-1}$. The interval for z variables is between 0 and 1, which guarantees ordering in k_+, and the angular ordering condition is satisfied by the constraint

$$\bar{q}_i > \bar{q}_{i-1} z_{i-1} \,, \tag{2.6}$$

explicitly written out in eq. (2.5). The distribution function depends on two separate scales, the transverse momentum k_T of the interacting gluon, and $\bar{q}$, which determines an angle beyond which there is no (quasi-) real parton in the chain of initial-state radiation. It may be argued [35] that the role of this variable is similar to that of μ^2 in the collinear gluon density.

The LDC model [18, 19] relies on the observation that the non-Sudakov form factor in equation (2.4) can be interpreted as a kind of Sudakov giving the no-emission probability in the region of integration. An additional constraint on the initial-state radiation is added requiring the transverse momentum of the emitted gluons to be above the smaller of the transverse momenta of the connecting propagating gluons:

$$q_{Ti} > \min(k_{Ti}, k_{Ti-1}) \,. \tag{2.7}$$

Emissions failing this cut will instead be treated as final-state emissions and need to be resummed in order not to change the total cross section. The remaining initial-state gluons are ordered both in q_+ and q_-, which implies that they are also ordered in angle or rapidity.

One single chain in the LDC model corresponds to a set of CCFM chains. It turns out that when one considers the contributions from all chains of this set, with their corresponding non-eikonal form factors, they add up to one [18]. Thus, the non-eikonal form factors do not appear explicitly in LDC, resulting in a simpler form for the unintegrated gluon distribution function:

$$\Phi(x, k_T^2) \sim \sum_n \int \prod^n \bar{\alpha} \frac{dz_i}{z_i} \frac{d^2 q_{Ti}}{\pi q_{Ti}^2} \theta(q_{+i-1} - q_{+i}) \times$$
$$\times \theta(q_{-i} - q_{-i-1}) \delta\left(x - \prod z_i\right) \delta(\ln k_T^2 - \ln k_{Tn}^2) \,. \tag{2.8}$$

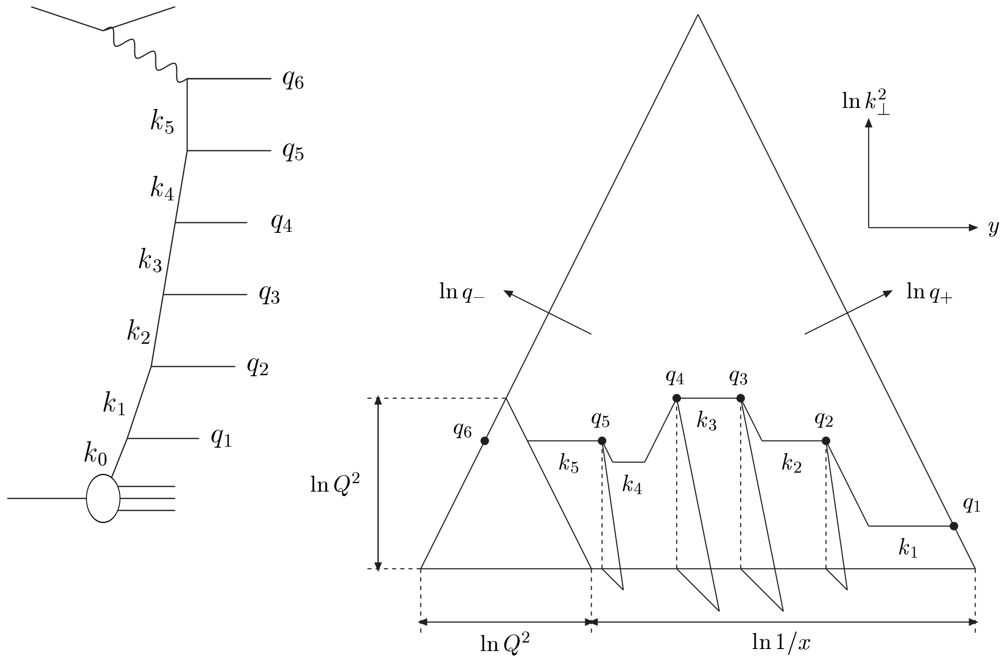

Figure 2: The initial-state emissions q_i in the $(y, \ln k_T^2)$-plane. Final state radiations is allowed in the region below the horizontal lines. The height of the horizontal lines determine $\ln k_{Ti}^2$.

The notation in (8) and what will follow refers to that of figure 2. Here, a typical DIS event is shown together with the corresponding phase space available in the $\gamma^* p$ rest frame, where the rapidity y and the transverse momentum q_T of any final-state parton are limited by a triangular region in the $(y, \ln q_T^2)$-plane. The proton direction is towards the right end of the triangle, and the photon direction is towards the left. The real emitted gluons are represented by points in this diagram. The virtual propagators do not have well defined rapidities, and are represented by horizontal lines, the left and right ends of which have the coordinates $(\ln k_{+i}/k_{Ti}, \ln k_{Ti}^2)$ and $(\ln -k_{Ti}/k_{-i}, \ln k_{Ti}^2)$ respectively. The phase space available for final-state emissions is given by area below the horizontal lines (including the fold that stick out of the main triangle).

The ordering of the CCFM evolution in q_+ but not q_- means that this formalism is not left-right symmetric. In contrast, the LDC formulation is completely symmetric, which implies that the chain in figure 2 can be thought of as evolved from either the photon or the proton end. Thus, the LDC formalism automatically takes into account resolved virtual photon contributions.

It is important to note that in the LDC model non-leading effects such as quark-initiated chains, the non-singular terms in the splitting functions and energy-momentum conservation can be included in a straight-forward manner. The fact that fewer gluons are considered as initial-state radiation implies that typical z-values are smaller, thus resulting in smaller sub-leading corrections. It is also implies that in the LDC formalism the gluon distributions is quite insensitive to the second scale $\bar{q}$, and to leading $\ln 1/x$ it depends only on a single scale k_T. Just as for CCFM, for finite x Sudakov form factors are introduced to regularize the $1/(1-z)$ poles in the splitting functions, thus introducing a $\bar{q}$ dependence.

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This dependence comes mainly from the last Sudakov form factor related to the convolution of the unintegrated parton densities with the off-shell matrix element, and the factorization

$$\Phi(x, k_T^2, \bar{q}^2) = \Delta_s(\bar{q}^2, k_T^2)\Phi(x, k_T^2) \tag{2.9}$$

is a good approximation for the LDC model.

3. Numerical results

In this section we present the numerical results of our calculations and compare them with data from DØ and CDF. We will use here the expressions for the heavy quark hadroproduction cross section and gluon-gluon fusion off-shell matrix element which were obtained in [26, 27].

To illuminate the effect of the different contributions we will study the four different versions of LDC unintegrated gluon distributions presented in [33], namely the *standard*, *gluonic*, *gluonic-2* and *leading* ones.

The *standard* version includes non-leading contributions from quarks and non-singular terms in the splitting functions. The *gluonic* and *leading* versions do not take into account quark links in the evolution. Furthermore the *leading* version does not includes non-singular terms in the splitting functions in contrast to the *gluonic* versions. For all versions, non-perturbative input parton densities of the form [36]

$$xf_i(x, k_{T0}^2) = A_i x^{a_i}(1-x)^{b_i} \tag{3.1}$$

where used, with all parameters A_i, a_i, b_i ($i = g, d, \bar{d}, u, \ldots$) and the perturbative cutoff k_{T0}^2 fitted [33] to reproduce the measured data on $F_2(x, Q^2)$. The *standard* version has been fitted in the region $x < 0.3$, $Q^2 > 1.5\,\mathrm{GeV}^2$ to experimental data taken by the H1 [37], ZEUS [38], NMC [39] and E665 [40] collaborations. The *gluonic* and *leading* versions have been fitted in the region $x < 0.013$ and $Q^2 > 3.5\,\mathrm{GeV}^2$ to data taken by the H1 collaboration only. To study the sensitivity to the b parameter in equation (3.1) an additional fit for *gluonic* case (called *gluonic-2*) was obtained (see [33] for detail information). We note that all versions give a satisfactory fit to the F_2 data.

After we fixed the choice of the unintegrated gluon distribution, our theoretical results depend on the choice of heavy quark mass value, factorization scale μ^2 and selection of the heavy quark fragmentation function. In the present paper we convert heavy quarks into D and B mesons using the usual Peterson fragmentation function [41] with $\epsilon = 0.06$ for charm and $\epsilon = 0.006$ for bottom respectively [42]. Also we used the following choice $m_c = 1.5\,\mathrm{GeV}$, $m_b = 4.75\,\mathrm{GeV}$ and $\mu^2 = m_T^2 = m_Q^2 + p_T^2$ with p_T being the transverse momentum of the heavy quarks in the $p\bar{p}$ c.m. frame. These choices are similar to the ones in e.g. [43].

The results of our calculations for bottom production are shown in figures 3–7. Figure 3 shows the prediction for the B^+ meson p_T spectrum for $|y_B| < 1$ at $\sqrt{s} = 1800\,\mathrm{GeV}$ compared to the CDF data [29]. One can see that results obtained with the *leading* version agree with the CDF data within experimental uncertainties. The *gluonic* version is very

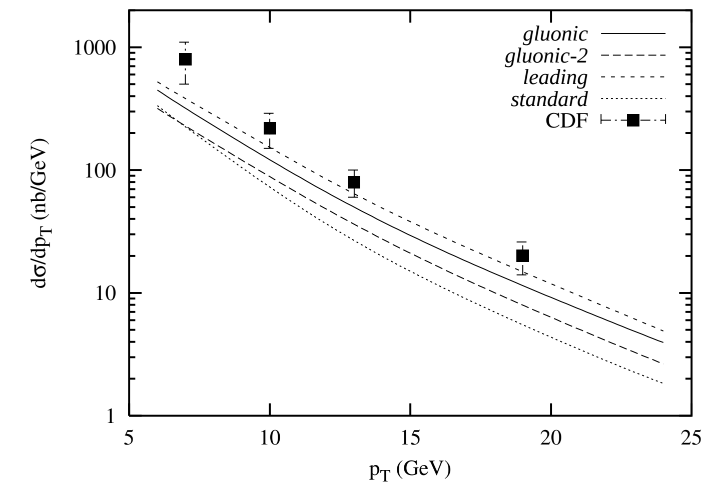

Figure 3: The prediction for the B meson p_T spectrum for $|y_B| < 1$ at $\sqrt{s} = 1800\,\mathrm{GeV}$ compared to the CDF data. The solid line is *gluonic*, dashed is *glionic-2*, short dashed is *leading* and dotted line is *standard* versions. Experimental data are from CDF [28].

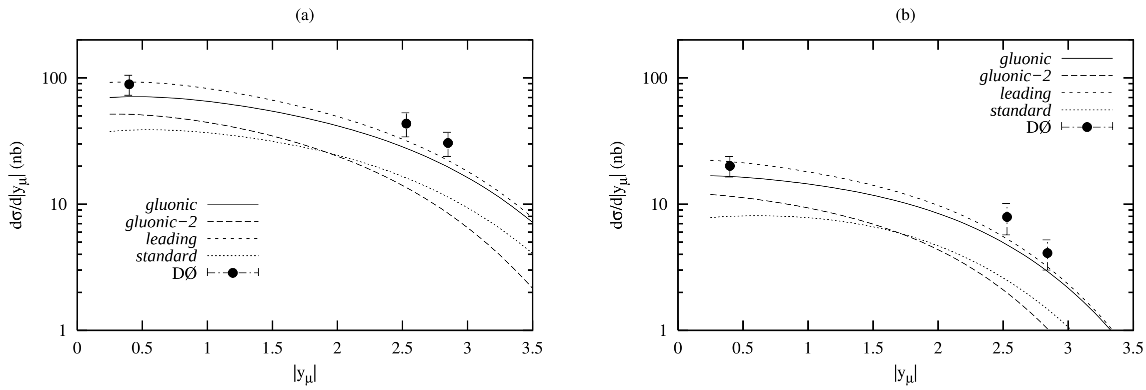

Figure 4: The cross section for muons with $p_{T\mu} > 5\,\mathrm{GeV}$ (a) and $p_{T\mu} > 8\,\mathrm{GeV}$ (b) from B meson decay as a function of rapidity compared to the DØ data. All curves are the same as figure 3. Experimental data are from DØ [32].

close to the *leading* one but goes a bit below the data. The results obtained using the *gluonic-2* and *standard* versions underestimate experimental data by a factor about 2 and are close to the NLO pQCD calculations [29]. One can also see a difference in the shapes between the *standard* version and the other ones, while the shapes of the *gluonic* and *leading* curves practically coincide.

The recent DØ [31, 32] experimental data refer also to muons which originate from the semileptonic decays of B-mesons. To produce muons from B mesons in theoretical calculations, we simulate their semileptonic decay according to the standard electroweak theory. Our calculation of the cross section for muons from B meson decay as a function of rapidity $d\sigma/d|y_\mu|$ is shown in figure 4 for both $p_{T\mu} > 5\,\mathrm{GeV}$ (4a) and $p_{T\mu} > 8\,\mathrm{GeV}$ (4b). We find that only *leading* and *gluonic* versions agree with the DØ experimental data [32] within errors. Again, the *leading* is somewhat above the *gluonic* although we now see that the

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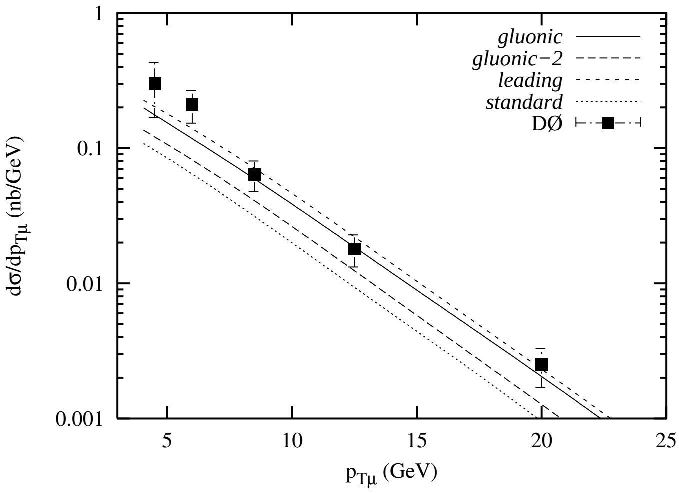

Figure 5: The leading muon p_T spectrum for $b\bar{b}$ production compared to the DØ data. The cuts applied to both muons are $4 < p_{T\mu} < 25\,\mathrm{GeV}$, $|\eta_\mu| < 0.8$ and $6 < m_{\mu\mu} < 35\,\mathrm{GeV}$. All curves are the same as in figure 3. Experimental data are from DØ [31].

difference due to the non-singular terms is mainly present in the central rapidity region. Also it is interesting to note that *standard* version have a more flat behavior compared to the other versions of the LDC unintegrated gluon distributions. The reason that the *gluonic-2* results falls much faster at large rapidities is that it has a more rapidly falling input distribution at large x ($b_g = 7$ in equation (3.1) rather than $b_g = 4$ as for the other versions). Note that the NLO pQCD calculations underestimate data by a factor about 4 [32].

Figure 5 shows the leading muon p_T spectrum in the central rapidity region for $b\bar{b}$ production compared to the DØ data [31]. The leading muon in the event is defined as the muon with highest transverse momentum. Again we find that the *leading* and *gluonic* versions agree well with the data while the other two are below. However, the shape of all curves practically coincide.

Also the double differential cross sections $d\sigma/dp_{T\mu}\,dy_\mu$ in the forward rapidity region $2.4 < |y_\mu| < 3.2$ are well described by the k_T-factorization approach with the *leading* and *gluonic* unintegrated gluon distributions (figure 6). Again there is only slight variations in the shape of the different cureves. The NLO pQCD calculations underestimate the DØ data [32] by a factor of 4.

In a previous paper [27] it was found that investigations of $b\bar{b}$ correlations such as the azimuthal opening angle between b and $\bar{b}$ quarks (or between their decay muons) is a powerful test for the different unintegrated gluon distributions. This is because these quantities are sensitive to the relative contributions of different production mechanisms to the total cross section [20, 21, 22, 23, 25]. In the naive collinear gluon-gluon fusion mechanism, the distribution over the azimuthal angle difference $\Delta\phi_{b\bar{b}}$ must be a simple delta function $\delta(\Delta\phi_{b\bar{b}} - \pi)$. Taking into account non-vanishing initial gluon transverse momenta leads to the violation of this back-to-back quark production kinematics in the k_T-factorization approach.

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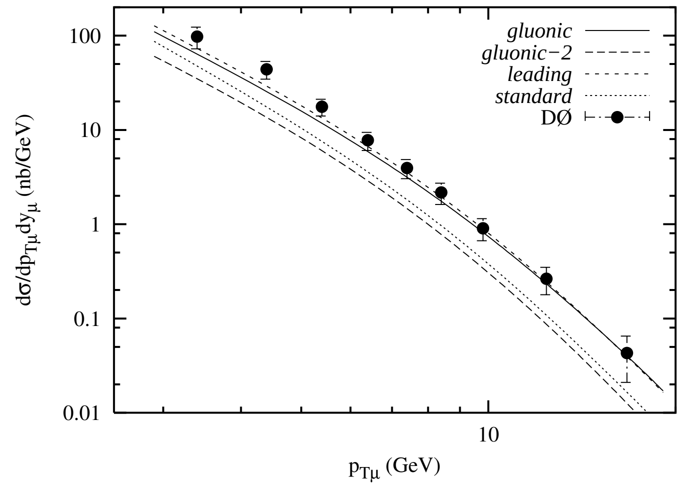

Figure 6: The double differential cross sections for muons from B meson decay with $2.4 < |y_\mu| < 3.2$ as a function of $p_{T\mu}$. All curves are the same as in figure 3. Experimental data are from DØ [32].

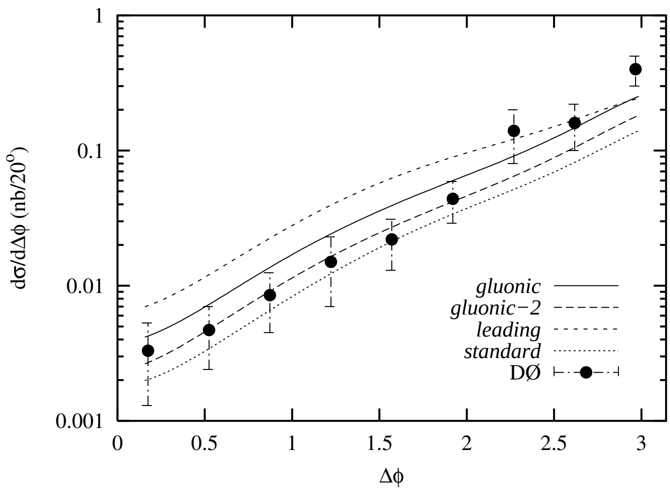

Figure 7: Azimuthal correlations for muons pairs with $4 < p_{T\mu} < 25\,\mathrm{GeV}$, $|\eta_\mu| < 0.8$ and $6 < m_{\mu\mu} < 35\,\mathrm{GeV}$. All curves are the same as in in figure 3. Experimental data are from DØ [31].

The differential $b\bar{b}$ cross section $d\sigma/d\Delta\phi_{\mu\mu}$ for central muons is shown in figure 7. For the overall cross section we se the same trend as in previous figures, where the *leading* is above the *gluonic* which in turn is above the *standard*. However, we here see from the shape that the *leading* clearly overestimates the decorrelation. The others curves agree better with the shape of the experimental result, although none of them are able to fully reproduce the peak at $\Delta\phi_{\mu\mu} \sim \pi$.

Very recently the CDF collaboration have reported preliminary experimental data [34] on charm production at the Tevatron Run II. These data found to be about a factor of 1.5 larger than NLO pQCD theoretical predictions [44]. data [34].

The results of our calculations of the transverse momentum spectra for centrally produced D^*, D^+ and D^0 mesons at $\sqrt{s} = 1960\,\mathrm{GeV}$ are shown in figure 8. Comparing with

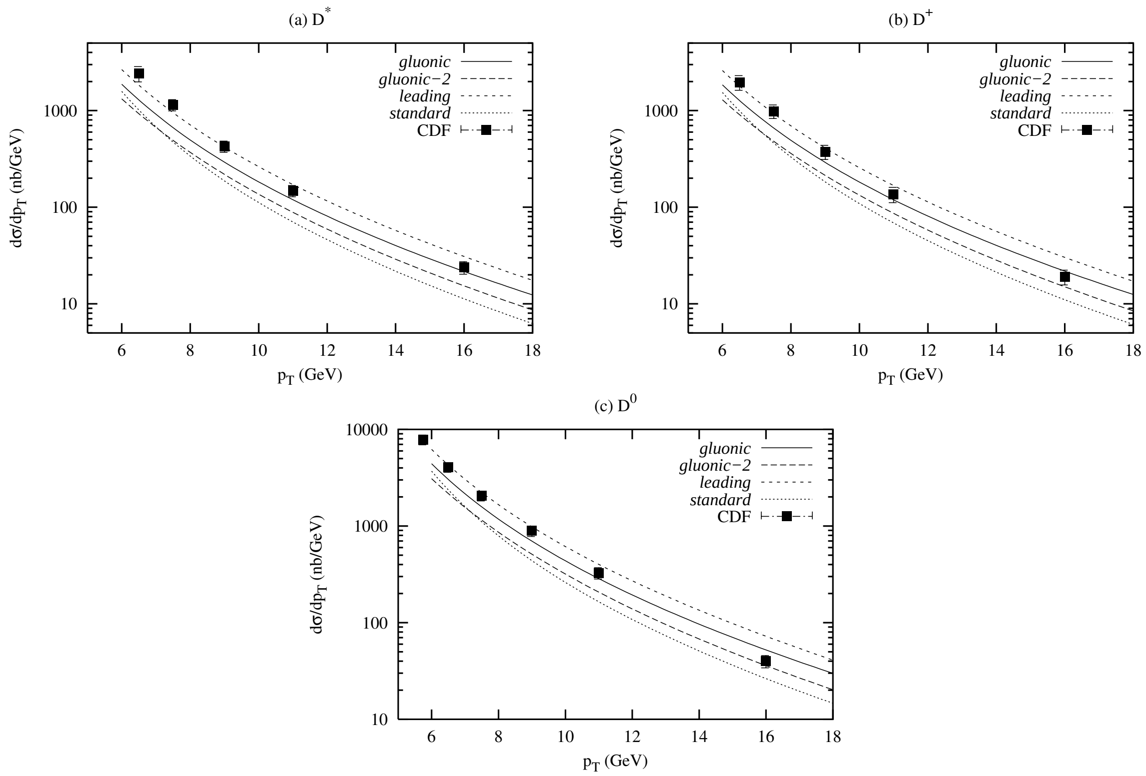

Figure 8: The prediction for the p_T spectrum of D^* (a), D^+ (b) and D^0 (c) mesons with $|y_D| < 1$ at $\sqrt{s} = 1960\,\mathrm{GeV}$ compared to the CDF data. All curves are the same as in figure 3. Experimental data are from CDF [34].

the B meson spectrum in figure 3, we see that the difference between the different versions of the LDC densities are the same both for the magnitude (i.e. *leading* > *gluonic* > *gluonic-2* > *standard*) and for the shape (*standard* falls more steeply than the others). However, the data is now a bit closer to the calculations, especially for the *leading* version.

4. Conclusions

We have presented results for charm and bottom production in $p\bar{p}$ collisions at high energies within the framework of k_T-factorization, using different unintegrated gluon densities obtained from the LDC model. It has previously been noted that the standard collinear factorization approach fails to describe the amount of heavy quarks produced at the Tevatron, and that k_T-factorization may be the more correct way of describing the underlying physics. Our results agrees with this observation. However, within the LDC model it is also possible to study non-leading effects in the evolution, and it is clear that these introduce large uncertainties. We have found that the inclusion of quarks in the evolution has a big effect and that the results are sensitive to the treatment of non-singular terms in the gluon splitting function. Also we have found that the results are sensitive to the assumed shape of the non-perturbative input density at large x.

In particular we find that we can only get an acceptable result for the case where only gluons are considered in the evolution and where only the singular terms in the gluon splitting function are included (*leading*). This is consistent with the findings in [24] where the CASCADE Monte Carlo [35] was shown to reproduce the amount of bottom production at the Tevatron. This program is also based on the CCFM equation and implements purely gluonic evolution without considering non-leading contributions in the splitting function. It is interesting to note that for the forward jet rates at HERA the situation is similar, as these can be reproduced by LDC and CASCADE only if non-singular terms are omitted [10].

Although the *leading* version of the LDC unintegrated gluon density gives a good description of the overall rate of heavy quark production, less inclusive observables such as azimuthal correlations between $b\bar{b}$ (or $\mu^+\mu^-$) pairs, are not as well described.

Besides the uncertainties due to non-leading terms in the evolution, there is, of course, also some uncertainties due to the chosen values of the heavy quark masses. However, the effects of the heavy quark masses would not be large enough to change the conclusions presented here. In addition, there is some freedom in the choice of factorization scale. We have here chosen $\mu = m_T$ which is the natural scale within the LDC model, but it can be noted that in e.g. [25], the scale was chosen to be $\mu = q_T$. Using such a scale in the LDC model would increase the bottom cross section, but, again, it would not be large enough to affect our conclusions.

It is important to note that we have here only considered gluon-gluon fusion into heavy quarks. One may also expect non-negligible contribution from two other production mechanisms, namely gluon splitting, where a hard final-state gluon splits into a heavy quark pair in a subsequent final-state cascade, and heavy quark excitation processes, where the the hard process is $gb \to gb$, with the b produced by initial-state gluon splitting earlier on in the evolution. In principle, all these contributions can be taken into account in the LDCMC Monte Carlo [36] which implements the LDC model, however, this program is not yet able to fully handle hadron-hadron collisions [45].

To conclude, we believe that k_T-factorization holds a possible key to understanding heavy quark production at the Tevatron. However, there are still large uncertainties, and much more work needs to be done before these processes are fully understood.

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